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Softly πĜB*- Normal Spaces

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Abstract

In this paper, we introduced a new concept of soft πĝb*-normality by using πĝb*-open set due to Nathiya and Vaiyomathi[**4**] . The concept of softly normal was introduced by M. C. Sharma and Hamant Kumar [**6**]. M. C. Sharma and Hamant Kumar [**6**] introduced a weaker version of normality called softly-normality and prove that soft-normality is a property, which is implied by quasi-normality and almost normality and obtained several properties of such a space. Recently, Nidhi Sharma and Neeraj Kumar Tomar [**5**] introduced a weaker version of normality called softly **Z*-**normality by using Z*-open set and obtained several properties of such a space. We introduced the concept of πĝb*-normal , almost πĝb*-normal, quasi πĝb*-normal, mildly πĝb*-normal .We prove the soft πĝb*-normality is a topological property and it is a hereditary property with respect to closed domain subspace. Moreover, we obtain some new characterizations and preservation theorems of softly πĝb* -normal spaces .We insure the existence of utility for new results of soft πĝb*-normality using separation axioms in topological spaces which is separate on a known separation axioms in topological spaces.

Keywords: π-closed, πĝb*-closed, α-closed sets, πĝb*-normal, almost πĝb*-normal, quasi πĝb*-normal, mildly πĝb*-normal, softly πĝb*-normal spaces.

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Introduction

We introduced a new class of softly normal called soft πĝb*-normality by using πĝb*-open set due to Nathiya and Vaiyomathi[4] and obtained several properties of such a space. We prove that soft πĝb*-normality is a topological property and it is a hereditary property with respect to closed domain subspace. Moreover, we obtain some new characterizations and preservation theorems of softly πĝb* -normal spaces

Aim of the Study

The aim of this paper, we introduced a new class of softly normal called soft πĝb*-normality by using πĝb*-open set due to Nathiya and Vaiyomathi[4] and obtained several properties of such a space. We introduced the concept of πĝb*-normal, almost πĝb*-normal, quasi πĝb*-normal, mildly πĝb*-normal .We insure the existence of utility for new results of soft πĝb*-normality using separation axioms in topological spaces which is separate on a known separation axioms in topological spaces.

Review of Literature

The concept of softly normal was introduced by M. C. Sharma and Hamant Kumar[6]. M. C. Sharma and Hamant Kumar[6] introduced a weaker version of normality called softly-normality and prove that soft-normality is a property, which is implied by quasi-normality and almost normality and obtained several properties of such a space. Recently, Nidhi Sharma and Neeraj Kumar Tomar [5] introduced a weaker version of normality called softly Z*-normality and prove that soft Z*-normality is a property, which is implied by quasi Z*-normality and almost Z*-nomality and obtained several properties of such a space.

Preliminaries

Definition

A subset A of a topological space X is called

1. α-closed [3] if $cl(int(cl(A))) \subseteq A$.
2. πg-closed [1] if $cl(A) \subset A$ whenever $A \subset U$ and U is π-open in X.
3. πĝb*-closed [4] if $int(b\text{-}cl(A)) \subset A$whenever $A \subset U$ and U is πg-open in X.
4. Regular closed [10] if $A = cl(int(A))$.
5. The finite union of regular open sets is said to be π-open.

The complement of α-closed (resp. πg-closed, πĝb*-closed, regular closed, π-closed) set is called α-open (resp. πg-closed , πĝb*-open, regular open, π-open) set. The intersection of all πĝb*-closed sets containing A is called the πĝb*-closure of A and denoted πĝb*-cl(A).The union of all πĝb*-open subsets of X which are contained in A is called the πĝb*-interior of A and denoted by πĝb*-int(A). Throughout this paper, (X, τ), (Y, σ) spaces always mean topological spaces X, Y respectively on which no separation axioms are assumed unless explicitly stated.

Definitions stated in preliminaries, we have the following diagram :

Closed ⇒ α-closed ⇒ πĝb*-closed

However the converses of the above are not true may be seen by the following examples.

Example

Let X = {a, b, c, d } and τ = { ϕ, {a}, {b}, {a, b}, X}.Then the set A = {b} is πĝb*-closed set but not closed set in X .

Example

Let X = {a, b, c, d } and τ = { ϕ, {a}, {b}, {a, b},{a, b, c}, X}.Then the set A = {c} is α-closed set as well as πĝb*-closed set but not closed set in X .

Remark

Every regular open (resp. regular closed) set is π-open (resp. π-closed).

Softly πĜB*- Normal Spaces

Definition

1. A topological space X is said to be Softly Normal [6](softly πĝb*-normal) if for any two disjoint subsets A and B of X, one of which is π-closed and other is regularly closed, there exist disjoint open(πĝb*-open) sets U and V of X such that A ⊂ U and B ⊂ V.
2. Almost-normal[7] (almost πĝb*-normal) if for every pair of disjoint sets A and B, one of which closed and other is regularly closed, there exist open (πĝb*- open) sets U and V of X such that A ⊂ U and B ⊂ V.
3. Quasi normal [11] (quasi πĝb*-normal) if for any two disjoint π-closed subsets A and B of X, there exist disjoint open (πĝb*-open) sets U and V of X such that A ⊂ U and B ⊂ V.
4. π-normal [2]](πĝb*-normal) if for any two disjoint closed subsets A and B of X, one of which is π-closed, there exist disjoint open (πĝb*-open) sets U and V of X such that A ⊂U and B ⊂V.
5. Mildly normal [8,9] (mildly πĝb*-normal) if for any two disjoint regularly closed subsets A and B of X, there exist disjoint open (πĝb*-open) sets U and V of X such that A ⊂ U and B ⊂ V.

By the definitions stated above, we have the following diagrams:

quasi normal ⇒ quasi πĝb*-normal ⇒ soft πĝb*-normal ⇒ mild πĝb*-normal

⇑

normal ⇒ π-normal ⇒ almost normal ⇒ softly normal ⇒ mildly normal

⇓ ⇓ ⇓ ⇓ ⇓

π-normal ⇒ πĝb*-normal ⇒ almost πĝb*-normal ⇒ soft πĝb*-normal ⇒ mild πĝb*-normal

Where none of the implications is reversible as can be seen from the following examples:

Example

Let X = { a, b, c, d } and τ = { ϕ, {a}, {c}, {a, c},{ b, d}, {a, b, d}, {b, c, d}, X }. The pair of disjoint π-closed subsets of X are A = {a} and B = {c}. Also U = {a} and V = {b, c, d} are disjoint open sets such that A ⊂ U and B ⊂ V. Hence X is quasi-normal as well as quasi πĝb*-normal as well as softly πĝb*-normal because every open set is πĝb*-open set.

Example

Let X = { a, b, c, d } and τ = { ϕ, {a}, { b}, {a, b}, {c, d},{a, c, d}, {b, c, d}, X }. Then A = {b} is closed and B = {a} is regularly closed sets there exist disjoint open sets U = {b, c, d} and V = { a} of X such that A ⊂ U and B ⊂ V. Hence X is almost normal as well as almost πĝb*-normal as well as softly πĝb*-normal because every open set is πĝb*-open set.

Example

Let X = {a, b, c, d} and τ = {ϕ, {b, d}, {a, b, d}, {b, c, d}, X}. The pair of disjoint closed subsets of X are A = {a} and B = {c}. Also U = {a, b} and V = {c, d} are πĝb*-open sets such that A ⊂ U and B ⊂ V. Hence X is πĝb*--normal but it is not normal.

Example

Let X = {a, b, c} and τ = {ϕ, {a}, {a, b}, {a, c}, X}. Then (X, τ) is almost -normal as well as almost πĝb*-normal, but it is not πĝb*-normal, since the pair of disjoint closed sets {b} and {c} have no disjoint πĝb*-open sets containing them. But it is not normal.

Example

Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, b, c}, {a, b, d}, X}. Then X is πĝb*-normal.

Theorem

For a topological space X, the following are equivalent:

a. X is softly πĝb*-normal.
b. For every π-closed set A and every regularly open set B with A ⊂ B, there exists a πĝb*-open set U such that A ⊂ U ⊂ πĝb*-cl (U) ⊂ B.
c. For every regularly closed set A and every π-open set B with A ⊂ B, there exists a πĝb*-open set U such that A ⊂ U ⊂ πĝb*-cl (U) ⊂ B.
d. For every pair consisting of disjoint sets A and B, one of which is π-closed and the other is regularly closed, there exist πĝb*-open sets U

and V such that A ⊂ U, B ⊂ V and πĝb*-cl (U) ∩ πĝb*-cl (V) = ϕ.

Proof

a. ⇒ (b). Assume (a). Let A be any π-closed set and B be any regularly open set such that A ⊂ B. Then A ∩ (X – B) = ϕ, where (X – B) is regularly closed. Then there exist disjoint πĝb*-open sets U and V such that A ⊂ U and (X – B) ⊂ V. Since U ∩ V= ϕ, then πĝb*-cl(U) ∩ V = ϕ. Thus πĝb*-cl (U) ⊂ (X – V) ⊂ (X – (X – B)) = B. Therefore, A ⊂ U ⊂ πĝb*-cl(U) ⊂ B.
b. ⇒ (c). Assume (b). Let A be any regularly closed set and B be any π-open set such that A ⊂ B. Then, (X – B) ⊂ (X – A), where (X – B) is π-closed and (X – A) is regularly open. Thus by (b), there exists a πĝb*-open set W such that (X – B) ⊂ W ⊂ πĝb*-cl (W) ⊂ (X – A). Thus A ⊂ (X – πĝb*-cl (W)) ⊂ (X – W) ⊂ B. So ,we let U = (X – πĝb*-cl(W)), which is πĝb*-open and since W ⊂ πĝb*-cl(W), then (X – πĝb*-cl(W) ⊂ (X – W). Thus U ⊂ (X – W), hence πĝb*-cl(U) ⊂ πĝb*-cl(X – W) = (X – W) ⊂ B.
c. ⇒ (d). Assume (c). Let A be any regular closed set and B be any π-closed set with A ∩ B = ϕ. Then A ⊂ (X – B), where (X – B) is π-open. By (c), there exists a πĝb*-open set U such that A ⊂ U ⊂ πĝb*-cl (U) ⊂ (X – B). Now, πĝb*-cl (U) is πĝb*-closed. Applying (c) again we get a πĝb*-open set W such that A ⊂ U ⊂ πĝb*-cl(U) ⊂ W ⊂ πĝb*-cl(W) ⊂ (X – B). Let V = (X – πĝb*-cl (W)), then V is πĝb*-open set and B ⊂ V. We have (X – πĝb*-cl (W)) ⊂ (X – W), hence V ⊂ (X – W), thus πĝb*-cl (V) ⊂ πĝb*-cl(X – W) = (X – W). So, we have πĝb*-cl(U) ⊂ W and πĝb*-cl(V) ⊂ (X–W).Therefore πĝb*-cl(U) ∩πĝb*-cl(V) = ϕ.⊂ d ⇒ (a) is clear.

Theorem

For a topological space X, the following are equivalent

a. X is softly πĝb*-normal.
b. For every pair of sets U and V, one of which is π-open and the other is regular open whose union is X, there exist πĝb*-closed sets G and H such that G ⊂ U, H ⊂ V and G ∪ H = X.
c. For every π-closed set A and every regular open set B containing A, there is a πĝb*-open set V such that A ⊂ V ⊂ πĝb*-cl(V) ⊂ B.

Proof

(a) ⇒ (b). Let U be a π-open set and V be a regular open set in a softly πĝb*-normal space X such that U ∪ V = X. Then (X – U) is π-closed set and (X – V) is regular closed set with (X – U) ∩ (X – V) = ϕ. By soft πĝb*-normality of X, there exist disjoint πĝb*-open sets U_1 and V_1 such that X – U ⊂ U_1 and X – V ⊂ V_1 . Let G = X – U_1 and H = X – V_1. Then G and H are πĝb*-closed sets such that G ⊂ U, H ⊂ V and G ∪ H = X. (b) ⇒ (c) and (c) ⇒ (a) are obvious.

Using Theorem 3.7**,** it is easy to show the following theorem, which is a Urysohn's Lemma version for soft πĝb*-normality. A proof can be established by a similar way of the normal case.

Theorem

A space X is softly πĝb*-normal if and only if for every pair of disjoint closed sets A and B, one of which is π-closed and other is regularly closed, there exists a continuous function f on X into [0, 1], with its usual topology, such that f(A) = {0} and f(B) = {1}.

It is easy to see that the inverse image of a regularly closed set under an open continuous function is regularly closed and the inverse image of a π-closed set under an open continuous function π-closed. We will use that in the next theorem.

Theorem

Let X is a softly πĝb*-normal space and f : X→ Y is an open continuous injective function. Then f(X) is a softly πĝb*-normal space.

Proof

Let A be any π-closed subset in f(X) and let B be any regularly closed subset in f(X) such that A ∩ B = ϕ. Then $f^{-1}(A)$ is a π-closed set in X, which is disjoint from the regularly closed set $f^{-1}(B)$. Since X is softly πĝb*-normal, there are two disjoint open sets U and V such that $f^{-1}(A)$ ⊂ U and $f^{-1}(B)$ ⊂ V. Since f is one-one and open, result follows.

Corollary

Soft πĝb*-normality is a topological property.

Lemma

Let M be a closed domain subspace of a space X. If A is a πĝb*-open set in X, then A ∩ M is πĝb*-open set in M.

Theorem

A closed domain subspace of a softly πĝb*-normal is softly πĝb*-normal.

Proof

Let M be a closed domain subspace of a softly πĝb*-normal space X. Let A and B be any disjoint closed sets in M such that A is regularly closed and B is π-closed. Then, A and B are disjoint closed sets in X such that A is regularly closed and B is π-closed in X. By soft πĝb*-normality of X, there exist disjoint πĝb*-open sets U and V of X such that A ⊂ U and B ⊂ V. By the above **Lemma,** we have U ∩ M and V ∩ M are disjoint πĝb*-open sets in M such that A ⊂ U ∩ M and B ⊂ V ∩ M. Hence, M is softly πĝb*-normal subspace.

Since every closed and open (clopen) subset is a closed domain, then we have the following corollary.

Corollary

Soft πĝb*-normality is a hereditary with respect to clopen subspaces.

Conclusion

In this paper, we have introduced weak form of normal space namely soft πĝb*-normality and established their relationships with some weak forms of normal spaces in topological spaces.

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